Game Theory

A review

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Game theory

- Key concept in multiagent systems
- Born to explain economical behaviour
- Assumption: Behavior of a given agent (player) interacting with other agents, usually with different objectives, is directed (again) to maximize the own profit
- The choice of action of one player has an impact on the other players' profit
- This is named an **strategic game**

Some definitions

- Strategic game: a scenario or situation where for two or more individuals their choice of action or behavior has an impact on the other (or others).
- Player: a participant in a strategic game.
- Strategy: a player's plan of action for the game (behavior).

Kinds of games

- One action game vs. extensive game
- Simultaneous actions vs. turn based games
- One shot vs. repeated games
- Cooperative vs. competitive
- Same information shared vs. different information
- Right now: one action, simultaneous, one time, same information games

Description of the game

- Pay-off: measures how well the player does in a possible outcome of a game.
- Each player has a pay-off function depending on the joint action chosen by the players
- This is described in the pay-off table

Pay-Off table for Agent 1

Agent 2

Agent 1

Action	a	b
a	α	β
b	γ	δ

Pay-Off table for Agent 2

Agent 2

Agent 1

Action	a	b
a	ε	κ
b	λ	o

Joint Pay-Off table

Agent 2

Agent 1

Action	a	b
a	α / ε	β / κ
b	γ / λ	δ / o

Example 1: Party-Club game

Person X

You

	party	club
party	-5 / 20	10 / -100
club	1 / 5	-10 / 1

Example 2: Penalty game

Goalkeeper

Messi

	left	centre	right
left	0 / 10	10 / 0	10 / 0
centre	10 / 0	0 / 10	10 / 0
right	10 / 0	10 / 0	0 / 10

[Constant sum games]

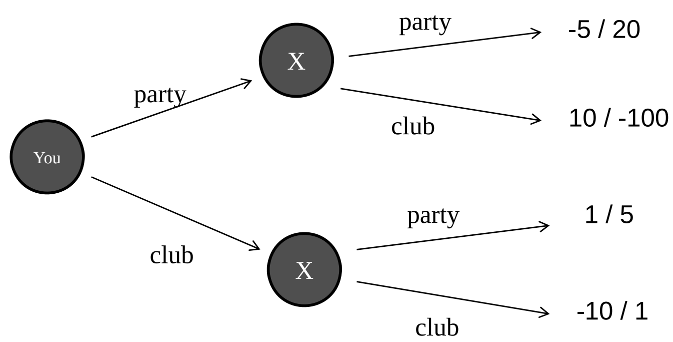
- **Constant-sum and zero-sum games:** Games in which the sum of the players' pay-offs is a constant. If the constant sum is zero the game is a zero-sum game. Constant-sum games are games of pure conflict; one player's gain is the other's loss.

Pay-Off table

- Only information needed to define the game
- Can be extended to *m* actions (even different actions and number of actions per player)
- Can be extended to *n* players

[In turn-based games]

- Representations of the game comes in the form of a trees (named extensive form or game tree)



Prediction of behavior

- Problem:
  - Given a pay-off table, which is the rational choice for each agent? (which is the strategic behavior of each agent?)
  - How can it be obtained?

Strategic behavior (policy!)

- Rational play: players choose strategies with the aim of maximizing their pay-offs

- Pure strategy: always the same action
- Mixed strategy: probabilistic mix of pure strategies

Mixed strategy

- Probabilistic mixture of deterministic strategies (policies)
- Take sense in iterated games
- Examples: Penalty game
- Mix of all three possible pure strategies (center, left and right)

Dominant strategy

- A strategy that is best response to *all* choices of other players

K

Q	s.offer	no offer
s.offer	10 / 14	18 / 6
no offer	4 / 20	7 / 8

Dominant strategy

- From the point of view of Q, to make an offer is better than no offer in any choice of K

K

Q	s.offer	no offer
s.offer	10	18
no offer	4	7

Dominant strategy

- So Q has a dominant strategy, and it is rational to assume that he will choose that one.

K

Q	s.offer	no offer
s.offer	**10**	**18**
no offer	4	7

Dominant strategy

- From the point of view of K…

K

	s.offer	no offer
Q: s.offer	14	6
Q: no offer	20	8

Dominant strategy

- It is better always to make an offer. So K has also a dominant strategy

K

	s.offer	no offer
Q: s.offer	**14**	6
Q: no offer	**20**	8

Strategic behavior (policy!)

- Dominant strategy: a 'best' strategy for a player in that it gives the player his highest pay-off given the strategy choices of all the players.
- Dominant strategy equilibrium in a game: a combination of players' strategies that are a best response to each other. Only happens when all players have a dominant strategy

Iterated dominance equilibrium

- What happens when both players do not have dominant strategies?
- Search for the iterated dominance equilibrium
- Possible equilibrium when at least one player present a dominant strategy

Iterated dominance equilibrium

- Game without dominance strategies for both players

X

You

	party	club
party	1 / 3	2 / 0
club	2 / 2	1 / 1

Iterated dominance equilibrium

- But player X has a dominant strategy

X

You

	party	club
party	**1 / 3**	2 / 0
club	**2 / 2**	1 / 1

Iterated dominance equilibrium

- You are sure that X will choose party
- So, you can remove club column

X

You

	party
party	**1 / 3**
club	**2 / 2**

Iterated dominance equilibrium

- Now, you have a dominant strategy

X

You

	party
party	1 / 3
club	**2 / 2**

Iterated dominance equilibrium

- {You:club, X:party} is the iterated dominance equilibrium
- Algorithm to find them:
  1. Check for a dominated strategy,
  2. Remove the dominated strategy
  3. Repeat the checking until only one strategy for player remains

Iterated dominance equilibrium

- Exercise

Mr.Row

Mr.Row		party	club	wedding
	party	1 / 3	2 / 0	1 / 1
	club	2 / 2	1 / 1	1 / 0
	wedding	1 / 3	1 / 4	0 / 5

Iterated dominance equilibrium

- Not all games without dominant strategy equilibrium have a iterated dominance equilibrium
- In some games there are more than one iterated dominance equilibrium!

Iterated dominance equilibrium

- More than one iterated dominance equilibrium, depending on the order of deletion

Mr.Row

Mr.Row		party	club	wedding
	party	3 / 6	2 / 6	1 / 7
	club	2 / 1	2 / 7	2 / 5
	wedding	3 / 7	1 / 5	2 / 6

Nash equilibrium

- Combination of players'actions (joint action) that are best responses to each other
- When all players play their Nash strategies, none of the players can do better

Nash equilibrium

- How to find the Nash Equilibrium?
- Find the best response

Mr.Row

Mr.Row	a	b	c
a	0 / 4	4 / 0	5 / 3
b	4 / 0	0 / 4	5 / 3
c	3 / 5	3 / 5	6 / 6

Nash equilibrium

- Underline the pay-off corresponding to the best response to each strategy of the other

Mr.Row

Mr.Row	a	b	c
a	0 / 4	4 / 0	5 / 3
b	4 / 0	0 / 4	5 / 3
c	3 / 5	3 / 5	6 / 6

Nash equilibrium

- Underline the pay-off corresponding to the best response to each strategy of the other

Mr.Row

Mr.Row	a	b	c
a	0 / 4	4 / 0	5 / 3
b	4 / 0	0 / 4	5 / 3
c	3 / 5	3 / 5	6 / 6

Nash equilibrium

- Underline the pay-off corresponding to the best response to each strategy of the other

Mr.Row

Mr.Row		a	b	c
	a	0 / 4	4 / 0	5 / 3
	b	4 / 0	0 / 4	5 / 3
	c	3 / 5	3 / 5	6 / 6

Nash equilibrium

- Underline the pay-off corresponding to the best response to each strategy of the other

Mr.Row

Mr.Row		a	b	c
	a	0 / 4	4 / 0	5 / 3
	b	4 / 0	0 / 4	5 / 3
	c	3 / 5	3 / 5	6 / 6

Nash equilibrium

- The joint action in which both coincide is the Nash equilibrium

Mr.Row

Mr.Row		a	b	c
	a	0 / 4	4 / 0	5 / 3
	b	4 / 0	0 / 4	5 / 3
	c	3 / 5	3 / 5	**6 / 6**

Nash equilibrium

- Exercise

K

Q		s.offer	no offer
	s.offer	10 / 14	18 / 6
	no offer	4 / 20	7 / 8

Nash equilibrium

- Every dominant strategy equilibrium and iterated-dominance equilibrium is a Nash equilibrium (but not necessarily the inverse)
- Every game has a Nash Equilibrium (but not always with pure strategies)
- Some games have more then one Nash equilibrium

Application to interesting games

- Coordination games

Y

X

	raise price	lower price
rise	5 / 5	1 / 2
lower	2 / 1	2 / 2

Application to interesting games

- Two Nash equilibrium. Which is the actual outcome of the game?

Y

X

	raise price	lower price
rise	**5 / 5**	1 / 2
lower	2 / 1	**2 / 2**

Pareto dominance

- **Pareto efficiency:** In a two-player game an outcome is Pareto-efficient if it is not possible to improve one player's pay-off without at the same time lowering the pay-off of the other.
- A Pareto optimum joint action is a Pareto efficient joint action
- When several equilibriums choose a Pareto efficient equilibrium…

Application to interesting games

- Pure coordination game

Y

X		raise price	lower price
	rise	**5 / 5**	1 / 2
	lower	2 / 1	**5 / 5**

Coordination

- In problems with several pareto optimum, there appear the problem of coordination.
- Which is the best strategy in the previous case?
- Usually solved by previously sharing information

Application to interesting games

- But… Let's meet the Prisoners' dilemma

Y

X		deny	confess
	deny	-1 / -1	-10 / 0
	confess	0 / -10	-5 / -5

Application to interesting games

- Is Nash equilibrium rational?

Y

X		deny	confess
	deny	-1 / -1	-10 / 0
	confess	0 / -10	**-5 / -5**

Prisoners' dilemma

- {Confess, confess} is pareto dominated by {deny, deny}.
- This could be solved as in coordination games by previously sharing information (previous agreement)
- But following the agreement would be not rational in this case! (you can increase the profit by not following it)

Pareto dominance

- **Pareto efficiency:** In a two-player game an outcome is Pareto-efficient if it is not possible to improve one player's pay-off without at the same time lowering the pay-off of the other.
- When several equilibriums choose a Pareto efficient equilibrium…
- But pareto optimal joint actions some times are not Nash equilibrium (prisoner's dilemma)

Pareto optimal joint action

- There can be several of them (coordination problem)
- There can be some of them more interesting for one player and other more interesting for the other
- Can only be found only by comparing *all* joint actions

Relation with Reinforcement Learning

- Interactions among agents will be modelized using game theory
- Pay-off table is the reward function
- Strategic behavior is the policy!
- **Model (pay-off table) is not known "a priori" by the agents**
- The game is repeated several times
- Which algorithms work in which cases?